

Classifying gapped phases of 4d gauge theories via temporal gauging of 1-form symmetry

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Wilson - 't Hooft classification

We consider 4d QFTs with $\mathbb{Z}_N^{[1]}$ symmetry (e.g. 4d $SU(N)$ YM + Adj matters), and classify their gapped vacua as quantum phases of matters.

Wilson - 't Hooft: Use line operators as order parameters.

$$\langle L(C) \rangle \sim \begin{cases} e^{-\sigma \cdot \text{Area}(C)} & (\text{confined}) \\ e^{-\mu \cdot \text{Length}(C)} & (\text{screened}) \end{cases}$$

"Line" operators in 4d gauge theories:

$$\begin{cases} W_{(0,1)}(C) (= W(C)) & \text{Wilson loop (electric)} \\ W_{(1,0)}(C) (= H(C)) & \text{'t Hooft loop (magnetic)} \end{cases}$$

$\Rightarrow$ We can classify the gapped states by specifying

$$\underline{H} \subset \tilde{\mathbb{Z}}_N \times \mathbb{Z}_N.$$

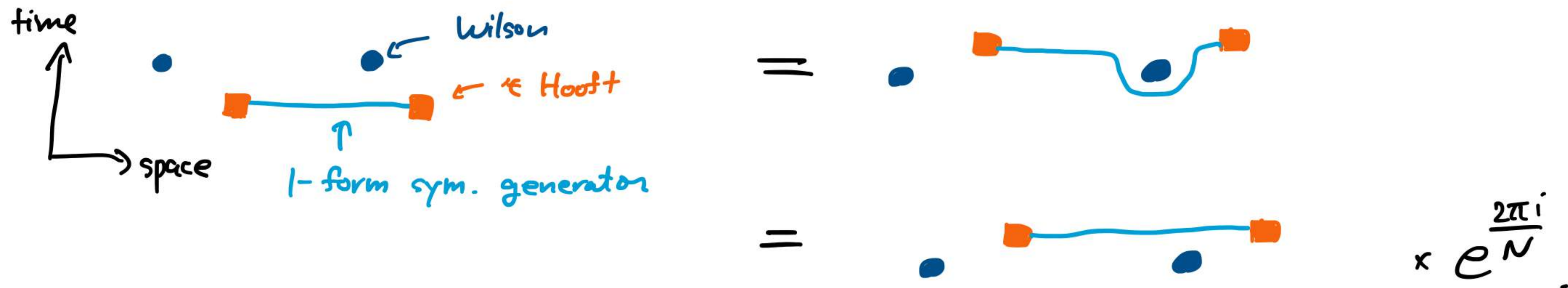
set of screened lines

Genuine line operators

We note that Wilson & 't Hooft line operators do not commute

$$\hat{W}(C) \hat{H}(C') = \hat{H}(C') \hat{W}(C) \cdot \exp\left(\frac{2\pi i}{N} \text{Link}(C, C')\right).$$

In a modern viewpoint,



i.e. 't Hooft line is not a genuine line operators (Aharony, Seiberg, Tachikawa).

Questions

- Why do we need 't Hooft lines?

Does it even make sense to use non-genuine lines?

- If it makes sense, what is classified by 't Hooft lines?

Main result

We show that

$$\mathbb{Z}_N^{[1]} \xrightarrow{\text{SSB}} \mathbb{Z}_{\underline{n}}^{[1]} \text{ enriched with the level-}\underline{k}\text{ } \mathbb{Z}_{\underline{n}}^{[1]} \text{-SPT}$$

$$\stackrel{1:1}{\iff} \tilde{\mathbb{Z}}_N \times \mathbb{Z}_N \rightarrow \underline{H} = \left\{ x(\underline{n}, 0) + y(\underline{k}, -\frac{N}{\underline{n}}) \right\} \text{ in Wilson-'t Hooft classification.}$$

↑ Any order- N subgroup of $\tilde{\mathbb{Z}}_N \times \mathbb{Z}_N$ can be expressed in this way.

We can use the screening behavior of non-genuine dyonic lines to specify SPT.

(← Reminiscent of the Kennedy-Tasaki transformation of the Haldane gapped state)

example 4d pure YM + θ -term

W : always confined (at least in large- N)

$$\begin{array}{c} z \sim e^{-\frac{2\pi i}{N} \int \frac{1}{2} B(B)} \quad z \sim 1 \quad z \sim e^{\frac{2\pi i}{N} \int \frac{1}{2} P_2(B)} \\ \text{---} \xrightarrow{\quad} \theta \\ \text{HW}^{-1} \quad H \quad \text{HW} : \text{deconfined} \end{array}$$

Our idea

We consider the S^1 -compactified set up

$$M_3 \times S^1$$

$$\left(\underbrace{\text{size}(M_3)} \gg \underbrace{\text{size}(S^1)} \gg \underbrace{\frac{1}{\text{mass gap}}} \right).$$

We may consider

3d effective QFTs on M_3

No phase transition under compactification

Under this compactification,

$$(\mathbb{Z}_N^{[1]})_{4d} \Rightarrow (\mathbb{Z}_N^{[1]})_{3d} \times (\mathbb{Z}_N^{[0]})_{3d}.$$

$$B_{4d} = B_m + \underbrace{A}_{\uparrow} \wedge \frac{d\tau}{L}.$$

We consider the gauging of $(\mathbb{Z}_N^{[0]})_{3d}$. (Temporal gauging)

$$\mathcal{Z}_{\text{tH}}[\tilde{B}_e, B_m] = \int \mathcal{D}a \, e^{\frac{2\pi i}{N} \int_{3d} \tilde{B}_e \cup a} \, \mathcal{Z}\left[B_m + a \wedge \frac{d\tau}{L}\right]. \quad [\text{cf. 't Hooft, '79, '81}]$$

This 3d theory enjoys $\tilde{\mathbb{Z}}_N^{[1]} \times \mathbb{Z}_N^{[1]}$ symmetry!
magnetic electric

So, we can apply the Wilson-'t Hooft scheme in a naive way.

Classification as topological states $\overset{1:1}{\iff}$ Wilson-'t Hooft classification

Proof

$\mathbb{Z}_N^{[1]} \xrightarrow{\text{SSB}} \mathbb{Z}_{\underline{n}}^{[1]}$ enriched with the level- $\textcircled{k}$ $\mathbb{Z}_{\underline{n}}^{[1]}$ -SPT

$\mathbb{Z}_{N/n}$ -TQFT

$$\mathcal{Z}_{\text{eff}}[B_{4d}] = \frac{|H^0(M_4; \mathbb{Z}_{N/n})|}{|H^1(M_4; \mathbb{Z}_{N/n})|} \sum_{[b] \in H^2(M_4; \mathbb{Z}_{N/n})} \exp\left(\frac{2\pi i}{N/n} \int b \cup B_{4d}\right)$$

$$\times \exp\left(\frac{2\pi i k}{n} \int \frac{1}{2} P_2\left(\frac{B_{4d}}{N/n}\right)\right)$$

temporal gauging
 $B_{4d} = B_m + a \wedge \frac{d\tau}{L}$
 $\Downarrow \int \mathcal{D}a \exp\left(\frac{2\pi i}{N} \int a \cup \tilde{B}_e\right) \mathcal{Z}[B_m + a \frac{d\tau}{L}]$

$$\mathcal{Z}_{\text{tH}} = N^{\beta_1(M_3)-1} \cdot \int_N \left[n B_m, \frac{N}{n} \tilde{B}_e + k B_m \right]$$

$$\tilde{\mathbb{Z}}_N \times \mathbb{Z}_N \rightarrow H = \{x(\underline{n}, 0) + y(\textcircled{k}, -\frac{N}{\underline{n}})\}$$

(i.e. $W_{(0,n)}(C) = W^n(C)$ and $W_{(N/n,k)}(C) = H^{N/n}(C) W^k(C)$ are deconfined) ■

Formal S & T operations

We can apply formal S & T operations in the space of 4d QFTs with $\mathbb{Z}_N^{(1)} \text{sym}$:

$$\begin{cases} S: \mathbb{Z}[B_{4d}] \mapsto S \cdot \mathbb{Z}[B_{4d}] = \int \mathcal{D}b_{4d} \mathbb{Z}[b_{4d}] e^{\frac{2\pi i}{N} \int b_{4d} \cup B_{4d}} \\ T: \mathbb{Z}[B_{4d}] \mapsto T \cdot \mathbb{Z}[B_{4d}] = \mathbb{Z}[B_{4d}] e^{\frac{2\pi i}{N} \int \frac{1}{2} P_2(B_{4d})} \end{cases}$$

Correspondingly,

$$S \cdot \mathbb{Z}_{\text{tH}} \left[\begin{pmatrix} \tilde{B}_e \\ B_m \end{pmatrix} \right] = \mathbb{Z}_{\text{tH}} \left[\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} \tilde{B}_e \\ B_m \end{pmatrix} \right]$$

↖ electro-magnetic dual

$$T \cdot \mathbb{Z}_{\text{tH}} \left[\begin{pmatrix} \tilde{B}_e \\ B_m \end{pmatrix} \right] = \mathbb{Z}_{\text{tH}} \left[\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \tilde{B}_e \\ B_m \end{pmatrix} \right]$$

↖ Witten effect

These operations form an $SL(2, \mathbb{Z}_N)$ automorphism on $\tilde{\mathbb{Z}}_N \times \mathbb{Z}_N$, and

we can relate different gapped states specified by order- N subgroups H_1, H_2 when they induce an isomorphism $H_1 \xrightarrow{\sim} H_2$.

Applications to gapped vacua for $\mathcal{N}=1^*$ SYM

$$\mathcal{N}=1^* \text{ SYM} = \mathcal{N}=4 \text{ SYM} + \text{soft SUSY-breaking to } \mathcal{N}=1$$

$$= \text{YM} + 4 \text{ adj. Weyl} + 3 \text{ adj. cplx scalar } X_1, X_2, X_3$$

$$W = \frac{1}{g^2} \text{Tr}(X_1[X_2, X_3]) + \frac{m}{2g^2} \text{Tr}(X_1^2 + X_2^2 + X_3^2)$$

classical vacua $\frac{\partial W}{\partial X_i} = 0 \Rightarrow [X_i, X_j] = -m \epsilon_{ijk} X_k : \text{SU}(2) \text{-algebra.}$

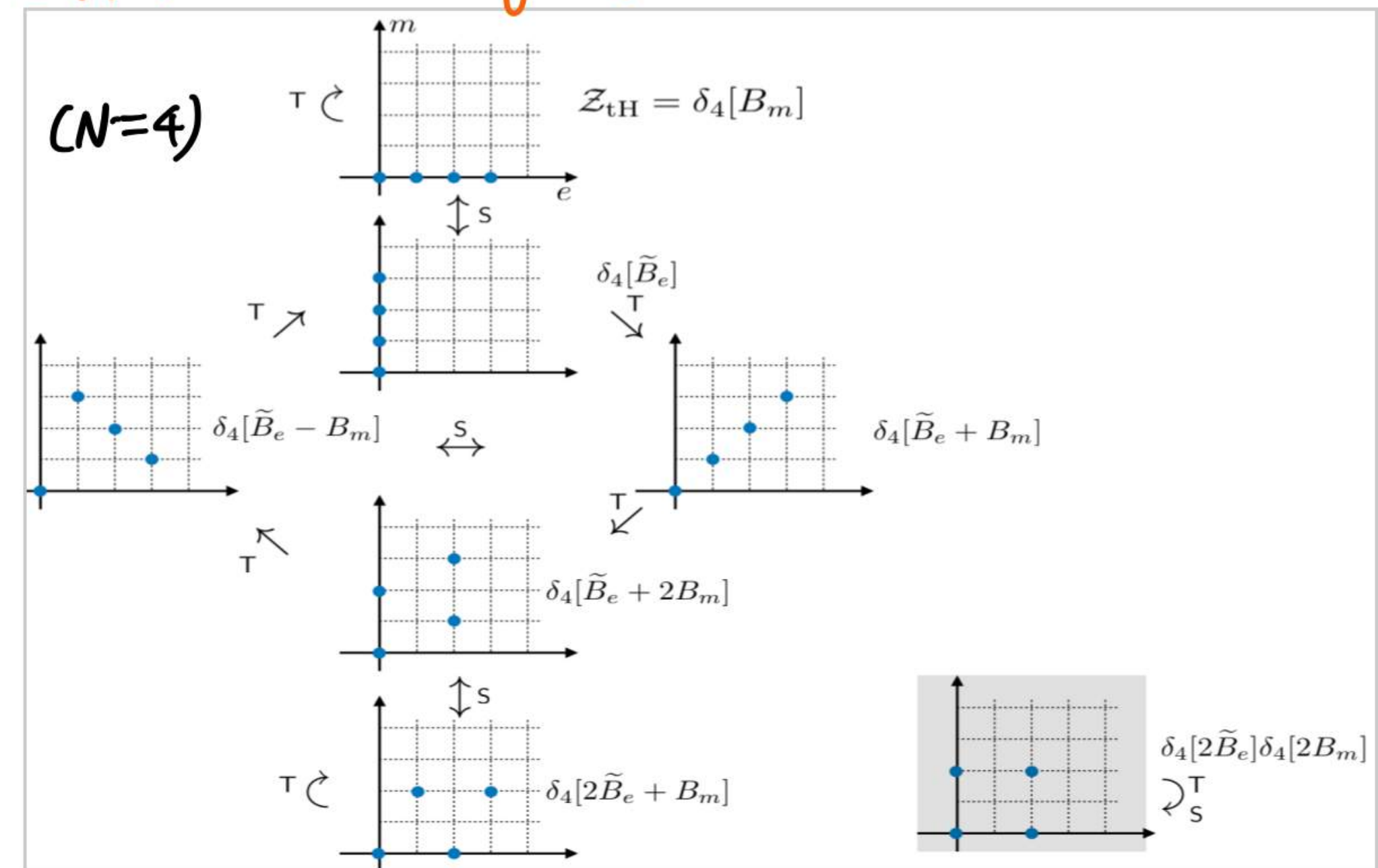
For $X_k = i m \mathbb{1}_n \otimes J_k^{(N/n)} \quad (k=1,2,3),$
local dynamics becomes
 $\mathcal{N}=1 \text{ SU}(n) \text{ SYM at low energies.}$

$\text{SU}(N) \xrightarrow{\text{Higgsing}} \frac{\text{SU}(n) \times \mathbb{Z}_N}{\mathbb{Z}_n}$

[cf. Donagi, Witten '96, Dorey '99]

$\mathcal{N}=1 \text{ SU}(n)$ is confining

$\Rightarrow$ Deconfined Wilson loop must be W^n ,
 which flows to the fundamental line of $\mathbb{Z}_{N/n}$ -TQFT.



Summary

- We relate two classification schemes for gapped phases of 4d QFTs w/ $\mathbb{Z}_N^{[1]}$ sym.

$$\mathbb{Z}_N^{[1]} \xrightarrow{\text{SSB}} \mathbb{Z}_{\underline{n}}^{[1]} \text{ enriched with the level-} \mathbb{k} \text{ } \mathbb{Z}_{\underline{n}}^{[1]} \text{-SPT}$$

$$\stackrel{1:1}{\iff} \tilde{\mathbb{Z}}_N \times \mathbb{Z}_N \rightarrow H = \left\{ x(\underline{n}, 0) + y(\mathbb{k}, -\frac{N}{\underline{n}}) \right\} \text{ in Wilson-'t Hooft classification.}$$

$$\mathbb{Z}_{\text{eff}}[B_{4d}] = \frac{|H^0(M_4; \mathbb{Z}_{N/n})|}{|H^1(M_4; \mathbb{Z}_{N/n})|} \sum_{[b] \in H^2(M_4; \mathbb{Z}_{N/n})} \exp\left(\frac{2\pi i}{N/n} \int b \cup B_{4d}\right) \\ \times \exp\left(\frac{2\pi i \mathbb{k}}{n} \int \frac{1}{2} P_2\left(\frac{B_{4d}}{N/n}\right)\right).$$

$$\begin{array}{l} \Downarrow \text{temporal gauging} \\ B_{4d} = B_m + a \wedge \frac{d\tau}{L} \\ \int D a \exp\left(\frac{2\pi i}{N} \int a \cup \tilde{B}_e\right) \mathbb{Z}[B_m + a \frac{d\tau}{L}] \end{array}$$

$$\mathbb{Z}_{\text{tH}} = N^{\beta_1(M_3)-1} \cdot \int_N [n B_m, \frac{N}{n} \tilde{B}_e + \mathbb{k} B_m]$$

- We consider the formal S & T operations in the space of 4d QFTs.
- We apply these results to 4d $N=1^*$ SYM theory.